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SYNTHESIS OF TWO-TERMINAL CONTACT-DIODE NETWORKS

by

E. J. Smith, C.M. Healy, and W. C. W. Mow

Research Report PIBMRI-1144-63

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ABSTRACT

An approach to the design of contact-diode networks having different forward and reverse transmission functions is described. Linear graph theory forms the basis for the synthesis procedure which is an extension of Gould's method. The properties of single-diode networks are considered first. For such networks, the two specified forward and reverse transmission functions are mapped onto a single-contact switching function. Next, an oriented circuit matrix is obtained and the synthesis of a graph corresponding to the matrix is attempted on a maximum-loop basis. If no graph exists, the number of columns in the matrix (correspondingly increasing the number of diodes or contacts) must be increased until one such matrix yields a realizable network.

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1. INTRODUCTION

It is well known that a switching function expressed as a Boolean polynomial is directly interpretable as a two-terminal series-parallel contact network, and that the addition of a bridging element to such a network often yields a switching function which would require a disproportionate number of contacts in series-parallel form. Moore's [1] table of minimal circuits of four variables contains a preponderance of non-series-parallel configurations. Accordingly, systematic network design procedures which are not prejudiced in favor of a particular network form have long been the objective of many research workers.

Certain design procedures which lead to non-series-parallel forms depend upon a special property of the switching function such as total or partial symmetry [2], [3]. Other procedures depend upon an assumed network form, such as the disjunctive tree [4], [5], [6]. There are two approaches to the design problem which are more general in character. The first is based upon the Boolean matrix [7, 8] and the second has its foundation in the theory of linear graphs [9], [10], [11]. Probably the first to apply graph theory to contact networks was Okada [9]. His contribution was in the synthesis of single-contact networks. The procedure has since been extended to non-single-contact networks by Gould [10] and Okada and Young [12].

The primary objective of the work done up to the present time on the application of graph theory to network design has been the determination of minimal forms. The procedure is essentially exhaustive in character and, consequently, the computations become unmanageable as the number of variables increases beyond five [13]. Although the minimal-contact solution is of considerable academic interest, it is often of little value due to the presence of switching hazards. Design procedures which yield near-minimal solutions and alternative-form solutions in a systematic way based upon graph theory appear to be a more realistic objective.

The purpose of the present paper is to explore the problems involved in the extension of the techniques of graph theory to two-terminal networks containing both diodes and contacts. Under these conditions the transmission function depends upon the direction of conduction in the source and detector. Examples are given in which the transmission functions in opposite directions through the terminals are specified. Various problems encountered along the way are examined and an innovation is described which leads in many cases to a simpler procedure for obtaining the network graph from its loop-set matrix.

2. SINGLE-CONTACT (S - C) SWITCHING FUNCTIONS

In this section we review briefly the relationship between a two-terminal bilateral contact network containing only one (make or break) contact per variable, and its associated graph.

Consider a topological graph G having n + 1 arcs and v vertices. Let one arc D be labled the distinguished arc, corresponding to a source and detector. The vertices incident upon D are the terminal vertices of the graph. With each of the remaining n arcs associate a binary switching variable x_i, and let each such variable be distinct and independent. Each vertex of G corresponds to a node of the contact network. The paths through D are related to the transmission function of the network in the following way. First, obtain the collection P of paths through the distinguished arc D; let each such path be denoted as ℓ_i. Next, form a term p_i which is the product of the elements in the path ℓ_i. Under these conditions, the sum of the product terms p_i (graph function) is in a one-to-one correspondence with the transmission function of the network. This function is called the S-C (single-contact) switching function F . Figure 1 shows a simple contact network and its corresponding graph. The collection of P of all paths through D is $\{a\ e,\ adfg,\ bcde,\ bcfg\}$ and the corresponding S - C function is F = ae + adfg + bcde + bcfg.

Synthesis of S - C (Graph) Functions:

The method for synthesizing a graph from a specified S - C switching function is based upon an important theorem [10] , [14] which states that a basis of any graph's loop-set vector space may be obtained by taking an independent set of the loop-set vectors derived from the S - C function. Thus the procedure follows in a straightforward way:

(1) The S -C function F is expressed in normal form as a sum of products in which no product term includes another product term.

(2) A loop-set matrix H is constructed from F. The columns of H correspond to the arcs in G; one column is assigned to each contact variable and to the distinguished arc D. The rows of H are the prime* loop-set vectors of the graph which contain D. Let $h_{ij} = 1$ if arc j is in path i and $h_{ij} = o$ otherwise.

(3) By elementary row operations (modulo 2) the matrix H is reduced to a fundamental loop-set matrix of the type.

$$L = [I\ T]$$

where I is the identity matrix and T is a submatrix corresponding to a tree of G.

*A prime loop-set vector is one which does not contain any other loop-set vector as a proper subset.

(4) A cut-set matrix K, orthogonal to L, is obtained from the relationship

$$L K^T = O \text{ modulo 2}$$

which requires that

$$K = \left[T^T \; I\right]$$

(5) By elementary row operations, K is reduced to a vertex cut-set matrix which contains no more than two 1's per column. A linearly dependent row is finally added giving the vertex-arc incidence matrix J from which the graph follows immediately.

(6) As a more effective alternative to steps (4) and (5), a graph can be determined directly [16] from the tree submatrix T of L in step (3). By still another method [12] a graph can be determined directly from the cut-set matrix K in step (4).

Synthesis of Non-S-C Functions

A transmission function in which each variable appears only in either the primed or unprimed form (i.e., a completely monotone function) will not, in general, correspond to an S-C function having the same number of variables; a suitable graph might require two or more contacts per variable. An arbitrary switching function will generally have a number of normal-form expansions. It is not known in advance which of these, if any, corresponds to an S-C function.* Thus the basic problem is to map a given switching function onto an acceptable S - C function.

Gould's method may be summarized as follows:

From the given switching function, a particular set of prime implicants is selected. Assuming tentatively that the function is indeed an S - C function, a loop-set matrix H is formed with a set of prime implicants selected as its loop-set vectors. The matrix H is tested to see if the space spanned by it is acceptable; that is, linear combinations of rows which yield a loop through D are formed, and in each case, the resulting path is examined to see if it is included in the function. If all such paths are included in the function, the vector space spanned by H is acceptable, and an attempt is made to realize a graph from it. If no graph is found, then other normal forms of the function involving no more contacts must be tried to see if one of them corresponds to an S-C function. Alternatively, row loop-set vectors, which do not result in additional loops through D, might be added in order to increase the dimensions of the vector space. These possibilities must be examined to determine if a graph results. If none of the k-column matrices leads to a solution, or if the vector space corresponding to H was found to be initially unacceptable, then matrices of (k + 1) columns must be examined. Such a matrix may be formed by splitting a column corresponding to some variable x_i in such a way that the new matrix is acceptable.

* The existence problem for Boolean branch-networks with only one branch for each different literal of the corresponding Boolean form has been investigated by L. Lofgren [17].

The process is continued until a solution is reached.

In a later section, an innovation is presented which offers some improvement in the procedure for examining H and in testing L for the existence of a special but relatively broad class of graph.

3. CIRCUITS CONTAINING DIODES

Consider a graph G having n + 1 arcs $\{x_1, x_2, x_3, \ldots, x_n\}$ and v vertices $\{a, b, c, \ldots, v\}$. Let D (a, b) be the distinguished branch and the vertices a, b be the terminal vertices of the graph. Let the arcs $x_1, x_2, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n$ represent bilateral contact elements. The branch x_i (1, 2) is taken to be a diode which conducts from vertex 1 to vertex 2 and represents an open circuit from vertex 2 to vertex 1. We wish to relate the topological paths in G through arc D to the corresponding electrical paths through the network. There are several types of paths to be considered. These are enumerated below:

I. Paths from a to b through D which include no more than one of the vertices (1, 2); then

$$p_{ab} = x_1 (a, \alpha)\, x_2 (\alpha, \beta) \ldots x_n (\xi, b)\, D (b, a)$$

and the transmission from a to b is

$$t_{ab} = x_1 x_2 \cdots x_n .$$

Also,

$$p_{ba} = x_n (b, \xi) \ldots x_2 (\beta, \alpha)\, x_1 (\alpha, a)\, D (a, b)$$

and

$$t_{ba} = x_1 x_2 \cdots x_n$$

thus

$$t_{ab} = t_{ba} \qquad \text{(Type 1)}$$

II. Paths from a to b through diode r (1, 2) = x_i (1, 2)

(i) Paths in which the vertices can be ordered

(a, -)...(1, 2)... (-, b) (b, a)

$$p_{ab} = x_1 (a, -) \ldots x_{i-1} (-, 1)\, r (1, 2)\, x_{i+1} (2, -) \ldots x_n (-, b)\, D (b, a)$$

$$t_{ab} = x_1 \cdots x_{i-1}\, x_{i+1} \cdots x_n$$

$$p_{ba} = x_n (b, -) \ldots x_{i+1} (-, 2)\, r (2, 1)\, x_{i-1} (1, -) \ldots x_1 (-, a)\, D (a, b)$$

$$t_{ba} = 0$$

(ii) Paths in which the vertices can be ordered

(a, -)... (2, 1)... (-, b)(b, a)

By similarity with (i)

$t_{ab} = O$ and $t_{ba} = x_1 \cdots x_{i-1}\, x_{i+1} \cdots x_n$

III. Paths from a to b which include both vertices (1, 2) but not branch r (1, 2)

(i) Paths in which the vertices can be ordered

(a, -)... (-, 1)... (2, -)... (-, b) (b, a)

then

$p_{ab} = x_1(a-) \cdots x_j(-, 1)\, x_{j+1}\, (1, -) \cdots x_{j+k}(2, -) \cdots x_n(-, b)\, D\, (b, a)$

$t_{ab} = x_1 \cdots x_j \cdots x_{j+k} \cdots x_n$

$p_{ba} = x_n\, (b, -) \cdots x_{j+k}\, (-, 2) \cdots x_{j+1}(-, 1)\, x_j\, (1, -) \cdots x_1(-a)\, D\, (a, b)$

$t_{ba} = x_1 \cdots x_j\, x_{j+1} \cdots x_{j+k} \cdots x_n$

therefore, $t_{ab} \supset t_{ba}$ (Type 2)

(ii) Paths in which the vertices can be ordered

(a, -)... (-, 2)... (1, -)... (-, b) (b, a)

By similarity with (i)

$t_{ab} = x_1 \cdots x_j\, x_{j+1} \cdots x_{j+k} \cdots x_n$

$t_{ba} = x_1 \cdots x_j \cdots x_{j+k} \cdots x_n$

therefore, $t_{ab} \subset t_{ba}$ (Type 3)

The transmission of such a network may be written as $T = \{T_{ab},\ T_{ba}\}$

$$T_{ab} = \sum t_{ab}\ (\text{Type 1}) + \sum t_{ab}\ (\text{Type 2}) + \sum t_{ab}\ (\text{Type 3})$$

$$T_{ba} = \sum t_{ba}\ (\text{Type 1}) + \sum t_{ba}\ (\text{Type 2}) + \sum t_{ba}\ (\text{Type 3})$$

where the summation is taken over all paths through D (a, b). We wish now to reverse the problem. That is, given a specified transmission function as shown above, find the contact network.

Networks Containing a Single Diode

If a network is realizable with a single diode and one contact per variable, then it is possible to obtain a consistent unoriented S-C function F from the given transmission function. Since for each product in the transmission function:

$t_{ab}=t_{ba}$ (Type 1); $t_{ab} \supset t_{ba}$ (Type 2), $t_{ab} \subset t_{ba}$ (Type 3),

the single-contact transmission is expressed as

$$F = \sum\left[t_{ab}(\text{Type 1}) + t_{ab}(\text{Type 3}) + t_{ba}(\text{Type 2})\right] + \sum\left[t_{ab}\ (\text{Type 2})\right]\ r\ (1,2) + \sum\left[t_{ba}\ (\text{Type 3})\right]\ r\ (2,1)$$

Matrix Representation of a Non-Bilateral Network:

Before attempting to synthesize a circuit for a non-bilateral switching function, it is necessary to consider a suitable matrix representation. As an example, we consider the circuit shown in Fig. 2a, having the switching function

$$T_{a\beta} = abc + def + aef + ace + cde$$

$$T_{\beta a} = abc + def + abf + bdf + bcd$$

From the non-oriented graph of the network (Fig. 2b) a loop-set matrix is obtained

$$L = \begin{array}{c} \\ \\ \\ \\ \end{array} \begin{array}{c} \begin{array}{ccccccccc} D & d & e & f & a & b & c & r_1 & r_2 \end{array} \\ \begin{bmatrix} 1 & . & . & . & 1 & 1 & 1 & . & . \\ . & 1 & . & . & 1 & . & . & 1 & . \\ . & . & 1 & . & . & 1 & . & 1 & 1 \\ . & . & . & 1 & . & . & 1 & . & 1 \end{bmatrix} \end{array} = \left[I\ T\right]$$

where T is the submatrix corresponding to the tree chosen in Fig. 2b. From this matrix we can generate all loops through D not shown explictly in L by taking the following combinations modulo 2.

$$H' = \begin{array}{c} \\ 12 \\ 13 \\ 14 \\ 123 \\ 124 \\ 134 \\ 1234 \end{array} \begin{array}{c} \begin{array}{ccccccccc} D & d & e & f & a & b & c & r_1 & r_2 \end{array} \\ \begin{bmatrix} 1 & 1 & . & . & . & 1 & 1 & 1 & . \\ 1 & . & 1 & . & 1 & . & 1 & 1 & 1 \\ 1 & . & . & 1 & 1 & 1 & . & . & 1 \\ 1 & 1 & 1 & . & . & . & 1 & . & 1 \\ 1 & 1 & . & 1 & . & 1 & . & 1 & 1 \\ 1 & . & 1 & 1 & 1 & . & . & 1 & . \\ 1 & 1 & 1 & 1 & . & . & . & . & . \end{bmatrix} \end{array}$$

The matrix modulo 2 does not convey enough information about the network. Therefore, an orientation must be assigned to the branches. For the orientation chosen in Fig. 2 c, the oriented loop-set matrix is obtained

$$L_o = \begin{array}{c} \begin{matrix} D & d & e & f & a & b & c & r_1 & r_2 \end{matrix} \\ \begin{bmatrix} +1 & . & . & . & +1 & +1 & +1 & . & . \\ . & +1 & . & . & -1 & . & . & -1 & . \\ . & . & +1 & . & . & -1 & . & +1 & -1 \\ . & . & . & +1 & . & . & -1 & . & -1 \end{bmatrix} \end{array}$$

The positive direction of orientation of arcs r_1 and r_2 was chosen to coincide with the direction of conduction through the rectifiers, although it is not necessary to do so. All loops through arc D can now be obtained by taking linear combinations of the rows of L_o in the real field such that each entry in the resulting matrix is either O, +1, or -1.

$$H^1_o = \begin{array}{c} \begin{matrix} D & d & e & f & a & b & c & r_1 & r_2 \end{matrix} \\ \begin{bmatrix} +1 & +1 & . & . & . & +1 & +1 & -1 & . \\ +1 & . & +1 & . & +1 & . & +1 & +1 & +1 \\ +1 & . & . & +1 & +1 & +1 & . & . & -1 \\ +1 & +1 & +1 & . & . & . & +1 & . & +1 \\ +1 & +1 & . & +1 & . & +1 & . & -1 & -1 \\ +1 & . & +1 & +1 & +1 & . & . & +1 & . \\ +1 & +1 & +1 & +1 & . & . & . & . & . \end{bmatrix} \end{array}$$

Transmission $T_{\alpha\beta}$ corresponds to loops oriented in the + direction through $D(\beta, \alpha)$. Consequently, we see that the first, third, and fifth rows of H^1_o represent non-conducting paths from α to β of the graph since, in each, either r_1 or r_2 has a negative sign. Correspondingly, if we multiply all rows of the matrix by -1, the second, fourth, and sixth rows represent non-conducting paths with respect to transmission from β to α.

In the synthesis procedure to be described, the technique of Gould is employed in part to obtain an acceptable loop-set matrix. However, since his problem dealt with bilateral elements only, there was no need for orienting the loop-set matrix. Thus the matrix obtained initially will have no orientation. It remains to convert a loop-set matrix modulo 2 into an oriented matrix containing elements from the real field, in such manner that the sign convention is consistent with the original matrix.

Orienting the matrix:

A matrix $L = [I\ T]$ modulo 2 can be replaced by a matrix containing elements +1, -1, and O, keeping the ranks of all submatrices invariant, if and only if no normal form of L contains either of the following submatrices [15].

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Furthermore, every oriented matrix L_o must be an E-matrix; that is to say, a matrix such that the determinant of every square nonsingular submatrix is either +1 or -1.

Since the elements of I in $L = [I\ T]$ appear only once, the sign of each is arbitrary and may be taken to be +1. In submatrix T, all non-zero elements of one column and one non-zero element in each of the remaining columns may be set equal to +1. Finally one non-zero element in each row may be set equal to + 1, provided that no non-zero element in that row had been previously given a + or - sign. The sign of the remaining non-zero entries of T might be determined by applying the previously stated theorem which requires that the determinate of every square submatrix in T be +1, -1, or 0. However, the execution of this procedure becomes unwieldly, except in the case of very small matrices. An alternative procedure is possible. The alternative is applicable to a class of L matrices, which nevertheless includes the great majority of switching circuits.

Maximum L Matrices:

We now return to the example circuit of Fig. 2a and choose a tree T described by the arcs $\{D, a, e, r_1, r_2\}$ as shown in Fig. 2d. If the arcs of the tree are given positive orientations with respect to the ordered path D a r_1 e r_2, then orientations for the links b, c, d, f can be found (Fig. 2d) such that each loop-set vector in the corresponding L matrix will contain only +1 non-zero entries; thus

$$L_M = \begin{array}{c} \begin{matrix} d & b & f & c & a & e & r_1 & r_2 & D \end{matrix} \\ \begin{bmatrix} +1 & . & . & . & +1 & . & +1 & . & . \\ . & +1 & . & . & . & +1 & +1 & +1 & . \\ . & . & +1 & . & +1 & +1 & +1 & . & +1 \\ . & . & . & +1 & +1 & +1 & +1 & +1 & +1 \end{bmatrix} \end{array}$$

The tree chosen in the previous example is a path which is included in a maximum loop of the graph. A maximum loop is defined as a prime loop passing through v vertices and containing v arcs.* It is easy to demonstrate that for any graph which contains a maximum loop, a maximum-loop matrix L_M can be found and that for such a matrix all non-zero elements can be set equal to +1; one row of the corresponding T submatrix must contain v - 1 arcs. All graphs such as those shown in Fig. 3 do not possess maximum loops. However, the graph of Fig. 3a can be converted into a maximum-loop graph by replacing one or both of its series-connected branches (a b or c d) by a single equivalent branch. Thus many graphs containing series-connected branches can be so converted into maximum loop graphs. For a class of non-oriented L matrices, the orientation process consists in reducing L to L_M by linear row operations modulo 2, and then setting all non-zero entries equal to + 1.

Realization of L_M Matrices:

Since the tree submatrix T_M for a given L_M corresponds to a maximum path containing all elements of T_M, the process of finding a graph, if one exists, is equivalent to the process of determining the appropriate ordering of the elements. The procedure is easy and can be carried out rapidly by direct inspection of T_M, or more formally by Goulds method [16]. If no graph exists for a given T_M, the ordering requirements for two or more rows are contradictory.

The ease with which an L_M can be realized suggests that it be adopted as the basis for synthesis of bilateral networks. Thus, for a given L corresponding to an acceptable non-oriented loop-set matrix H, the procedure is to convert L to an L_M by linear row operations. If L cannot be converted to an L_M, even by replacement of series-connected branches by a simple equivalent branch, then no graph exists or, if a graph exists, it is not in the class of graphs represented by L_M matrices. The restriction imposed by adopting L_M as a basis for synthesis would not appear to be unreasonable for graphs of switching circuits.

Synthesis of a Diode-Contact Network:

We have a method for converting a matrix modulo 2 into a matrix in the real field and of interpreting this oriented matrix to determine the various paths through the diodes. It remains to establish a procedure for obtaining a suitable loop-set matrix. The approach is outlined in the following:

1. We make the initial assumption that the network can be realized with one contact per variable and one diode. Accordingly, the S-C function F is obtained from the transmission function $\{T_{ab}, T_{ba}\}$, in which T_{ab} and T_{ba}

* In a graph, a path-tree is said to be Hamiltonian if it passes once and only once through every vertex of the graph; and a loop is said to be Hamiltonian if it passes once through every vertex of the Graph [18].

are in prime-implicant form. The loop-set matrix H is then constructed.

2. The loop-set matrix is tested for acceptability and, if the matrix contains unacceptable paths, contacts may be split. However, a diode may also be split to block an unacceptable path; if there are two diodes connected back-to-back in a path, then there is no electrical transmission between the terminals.

3. Once H has been made acceptable, the matrix is converted to an L_M matrix. If necessary, additional linearly independent rows corresponding to internal loops (i.e., not passing through D) may be added to H. If no L_M is possible, it is assumed that no graph exists. Other arrangements acceptable for splitting the contacts or another form of F must be examined until an L_M is obtained.

4. L_M is oriented immediately and the paths through D are checked. If the diode orientation is inconsistent with the transmission requirements the diode may be split to obtain a suitable set of oriented circuits.

5. Once the oriented matrix has been made acceptable, it is converted, if necessary, again to the maximum loop form L^1_M and an attempt is made to realize a graph from its tree submatrix.

6. If there is no graph corresponding to L^1_M, another form of contact or diode splitting must be tried and the attempt repeated.

Examples illustrating the various ideas will be presented in the following section.

4. EXAMPLES OF SYNTHESIS PROCEDURE

In this section we present first two examples involving bilateral switching functions in order to illustrate certain ideas related to the L_M matrix. This is followed by three examples involving non-bilateral functions. Since we have not improved the process of finding an acceptable H matrix, we do not dwell on this aspect of the problem. It should be assumed that the splitting of contacts, when necessary, is executed as in previous works [10] , [12] .

Example 1:

The switching function is given by the Karnaugh map shown in Fig. 4. The first three prime implicants in f cover the function and are chosen as a basis for the loop-set matrix H. H is acceptable since the modulo 2 sum of the three rows contains the term y'y and z'z which represents a non-conducting path through D. By interchanging columns, an L is obtained immediately, but it does not contain a maximum loop. However, we observe from L that y'z and wy appear as series elements. When each pair is represented as an equivalent matrix element, L_M is obtained. The graph is drawn by inspection. The result is minimal in contacts but not in springs.

The rank of H and L is seen to be 5; consequently, a row containing 6 ones is necessary, but the largest row in either matrix contains 4 ones. However, by appending a suitable row, representing an internal loop, the rank would be decreased by one and the number of ones required in some row would be 5. To be suitable, the appended row must not be a proper subset of another row; otherwise the second row could not be a prime loop set. Furthermore, the appended row when added to any D-row in the original H must yield a D-loop contained in f. Suitable rows in this example are wz' and wz'z, and both yield graphs as shown in Fig. 5. Other solutions are also possible.

Example 2:

The bilateral switching function is given in Fig. 6 and the loop-set matrix H_1 is set up from the irredundant prime implicants corresponding to f_1. The dependent row (123) describes the term wx which is not contained in the given function, and is therefore unacceptable. To obtain a solution, contacts w' or x' might be split in various ways. Alternatively, a loop-set matrix H_2 can be set up from the redundant functional form f_2. In this case, the dependent row (123) corresponds to the unacceptable term wxy. An acceptable loop-set matrix may now be obtained by splitting the y' contact into y_1' and y_2' in one of the three ways (a, b, or c) shown in the figure. Combinations (a) and (b) lead to non-realizable matrices; combination (c) leads to the maximum-loop matrix L_{M2_c} which is realized by the graph shown in Fig. 6, the result is minimal in contacts but not in springs.

Other solutions can be obtained by appending suitable rows corresponding to internal loops. Rows corresponding to wx' or w'x, for example, are suitable. The result obtained by adding a wx' row to L_{M2_c} (where w'x' and wx are now separated into individual elements w', x', w, x) is shown in Fig. 7. This solution is minimal in both contacts and springs[1].

Example 3:

The non-bilateral transmission function $\{T_{12}, T_{21}\}$ is given in Fig. 8. By comparing terms in T_{12} and T_{21}, we find:

Type 1 - abc, fg, cef
Type 2 - ag $\supset$ abeg, adg, and ace $\supset$ acde
Type 3 - bcf $\supset$ bcdf

The non-oriented S-C switching function is taken as F and the matrix H is formed. H is converted directly to L_M as shown. Since all D-loops in H are acceptable, details are omitted. L_M is, of course oriented immediately with positive signs (not shown) assigned to non-zero entries. Row 2 in L_M represents the transmission ag which belongs to T_{12}. Therefore, let the +D indicate terms in T_{12} and +1 indicate the conducting direction of the diode. All terms of interest including the diode must now be generated; they are rows (2-1) and (1 + 2 - 4 - 5) as shown. The first yields a

transmission path of ace in T_{12} and the second a transmission path of bcf in T_{21}. Both of these are consistent with the specified transmission function. Therefore, L_M is acceptable, and the graph is drawn directly from L_M as shown.

Example 4:

The loop-set matrix corresponding to the F function for the given transmission function is constructed as shown in Fig. 9. Rows 6 and 7 of H are linearly dependent and may be dropped. The unacceptable dependent rows obtained from H are (234), (235), and (245). We note, however, that if the diode is split into two diodes(r_1 and r_2) the unoriented matrix can be made acceptable. The allowable combinations of r_1 and r_2 are shown as (a), (b), and (c) in Fig. 9. If combination (a) is tried, H can be converted directly to L_{M1} in Fig. 10. Row 3 of L_{M1} corresponds to the term (ae) in T_{12}; thus let the plus sign for r_1 and r_2 indicate the direction of conduction and let +D indicate transmission from terminals 1 to 2 through the network. Other paths through r_1 and r_2 may be generated from L_{M1}. Among them is the path (124) or ($bcdr_1r_2$) which should belong to T_{21}, but the two diodes are connected back-to-back with r_1 blocking. Hence the oriented matrix is unacceptable.

If either combination (b) or (c) is chosen, the matrix L_{M2} results. Since row 1 represents the path abe which is in T_{21}, let +D represent the path from terminals 1 to 2, and let $+r_2$ represent the reverse biased direction of r_2. Similarly for row 3 let $+r_1$ indicate conduction and +D represent the path from 1 to 2 through the network. Then the term ae is in T_{12}. All other paths of interest are now generated as shown in Fig. 10. Row (123) is included in de. Row (24) indicates that cd is in T_{12}. Row (34) indicates that the term ac is in T_{12}. Finally, row (1234) shows that the term bcd is in T_{21}. L_{M2} is acceptable as an oriented loop-set matrix and the graph is drawn.

Example 5:

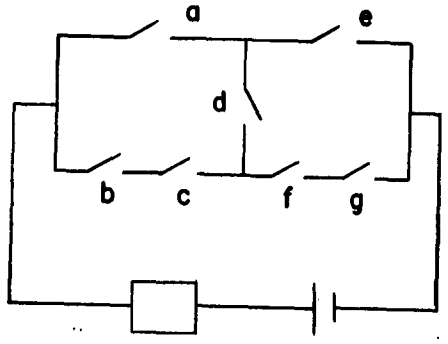
From the specified transmission function, the non-oriented matrix H is set up in the usual way as shown in Fig. 11. Row (123) corresponding to the term $xy'z$ is unacceptable. Two approaches are possible: split contact x', or split diode r. If x' is split, H is converted to L_1 as shown; if r is split, H can be converted to L_2 of Fig. 11. However, L_2 contains the submatrix

$$\begin{bmatrix} . & 1 & 1 & 1 \\ 1 & . & 1 & 1 \\ 1 & 1 & . & 1 \end{bmatrix}$$

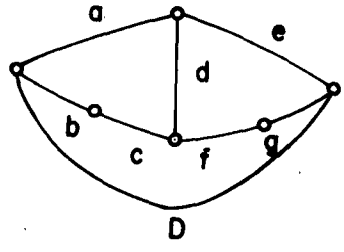
and so is unrealizable. L_1 is readily converted to a maximum loop-set matrix L_{M1}. Now, by inspection of the sign of r it is clear that either both of rows 1 and 2 (xyz' and x'yz) will be in T_{12} or in T_{21}, but that it is not possible to place one term in each transmission. Hence, L_{M1} is unacceptable. If, however, r is split into two diodes r_1 and r_2, then the orientation of each may be selected at will. Thus matrix L^*_{M1} is obtained. (L^*_{M1} is maximum loop if r_2 is considered as part of the x_1' z series branch.) In this result, $+r_1$ indicates the direction of conduction of r_1 and $+r_2$ is taken to be the reverse-biased direction of r_2. The graph follows immediately.

5. CONCLUDING REMARKS

This work represents an examination of the problems involved in the application of the theory of linear graphs to the synthesis of bilateral and non-bilateral contact networks. The objective is not to determine minimal circuits, but to search for techniques which might lead to better ways of obtaining economical circuits. The adoption of the maximum-loop matrix L_M from an acceptable H matrix as the basis for synthesis seems to have the advantage of increasing the speed with which alternative forms of H can be examined. The selection of an acceptable (and economical) H matrix in a straightforward way is yet unsolved. However, it would seem to be possible, with further development, to make use of the maximum-loop condition in the determination of acceptable split-contact arrangements.



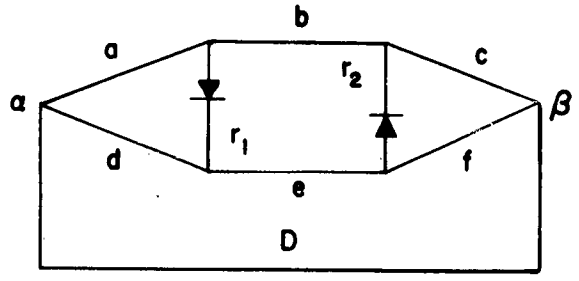
Detector Source



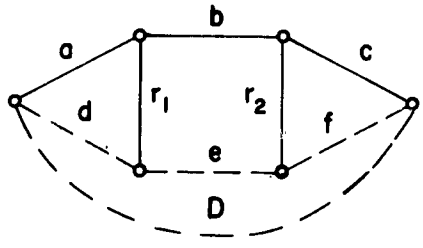
Paths through D: $\{ae, adfg, bcde, bcfg\}$

S–C Function: $F = ae + adfg + bcde + bcfg$

Fig. 1

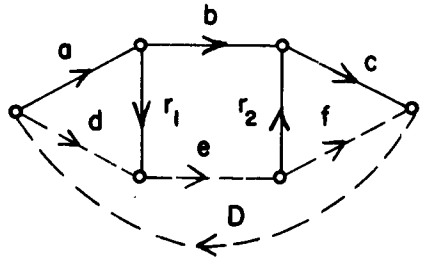


(a)



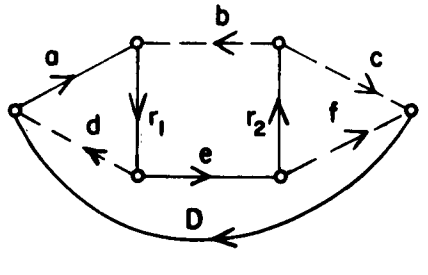
(b)

$$L = \begin{bmatrix} D & d & e & f & a & b & c & r_1 & r_2 \\ 1 & \cdot & \cdot & \cdot & 1 & 1 & 1 & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot & 1 & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & 1 & \cdot & \cdot & 1 & \cdot & 1 & 1 \\ \cdot & \cdot & \cdot & 1 & \cdot & \cdot & 1 & \cdot & 1 \end{bmatrix}$$



(c)

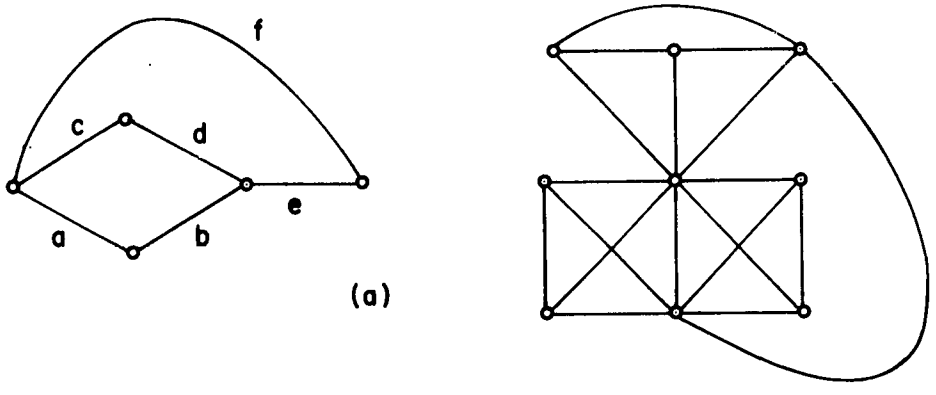
$$L_O = \begin{bmatrix} +1 & \cdot & \cdot & \cdot & +1 & +1 & +1 & \cdot & \cdot \\ \cdot & +1 & \cdot & \cdot & -1 & \cdot & \cdot & -1 & \cdot \\ \cdot & \cdot & +1 & \cdot & \cdot & -1 & \cdot & +1 & -1 \\ \cdot & \cdot & \cdot & +1 & \cdot & \cdot & -1 & \cdot & -1 \end{bmatrix}$$



(d)

$$L_M = \begin{bmatrix} d & b & f & c & a & e & r_1 & r_2 & D \\ +1 & \cdot & \cdot & \cdot & +1 & \cdot & +1 & \cdot & \cdot \\ \cdot & +1 & \cdot & \cdot & \cdot & +1 & +1 & +1 & \cdot \\ \cdot & \cdot & +1 & \cdot & +1 & +1 & +1 & \cdot & +1 \\ \cdot & \cdot & \cdot & +1 & +1 & +1 & +1 & +1 & +1 \end{bmatrix}$$

Fig. 2



Simple Non—Maximum—Loop Graphs

Fig. 3

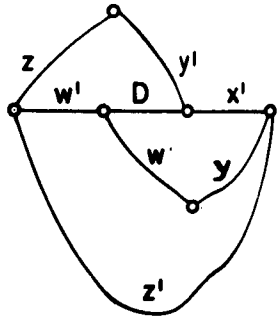
wx \ yz	00	01	11	10
00	1	1		1
01		1		
11				
10			1	1

$$f = w'x'z' + w'y'z + wx'y + (w'x'y') + (x'yz')$$

$$H = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 123 \end{array} \begin{array}{c} \begin{array}{cccccccc} w' & w & x' & y' & y & z' & z & D \end{array} \\ \begin{bmatrix} 1 & \cdot & 1 & \cdot & \cdot & 1 & \cdot & 1 \\ 1 & \cdot & \cdot & 1 & \cdot & \cdot & 1 & 1 \\ \cdot & 1 & 1 & \cdot & 1 & \cdot & \cdot & 1 \\ \cdot & 1 & \cdot & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$L = \begin{array}{c} \begin{array}{cccccccc} z' & z & w & w' & x' & y' & y & D \end{array} \\ \begin{bmatrix} 1 & \cdot & \cdot & 1 & 1 & \cdot & \cdot & 1 \\ \cdot & 1 & \cdot & 1 & \cdot & 1 & \cdot & 1 \\ \cdot & \cdot & 1 & \cdot & 1 & \cdot & 1 & 1 \end{bmatrix} \end{array}$$

$$L_M = \begin{array}{c} \begin{array}{cccccc} z' & (y'z) & (wy) & w' & x' & D \end{array} \\ \begin{bmatrix} 1 & \cdot & \cdot & 1 & 1 & 1 \\ \cdot & 1 & \cdot & 1 & \cdot & 1 \\ \cdot & \cdot & 1 & \cdot & 1 & 1 \end{bmatrix} \end{array}$$

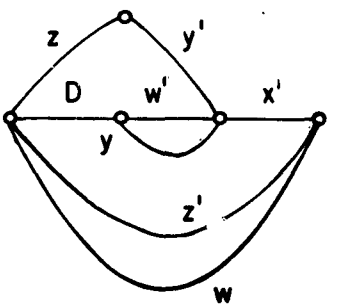


Example 1 — Fig. 4

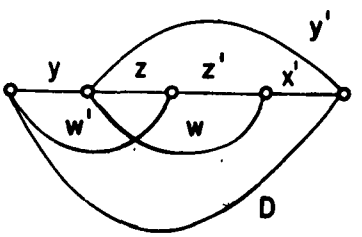
$$wz' = \begin{bmatrix} \cdot & \cdot & | & \cdot & \cdot & \cdot & | & \cdot \end{bmatrix}$$ Added to H gives L_{M2}

$$wzz' = \begin{bmatrix} \cdot & \cdot & | & \cdot & \cdot & \cdot & | & | \end{bmatrix}$$ Added to H gives L_{M3}

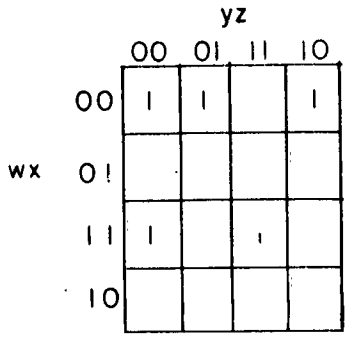
$$L_{M2} = \begin{array}{c} \begin{matrix} z' & (zy') & y & w & w' & x' & D \end{matrix} \\ \begin{bmatrix} | & \cdot & \cdot & \cdot & | & | & | \\ \cdot & | & \cdot & \cdot & | & \cdot & | \\ \cdot & \cdot & | & \cdot & | & \cdot & \cdot \\ \cdot & \cdot & \cdot & | & | & | & | \end{bmatrix} \end{array}$$



$$L_{M3} = \begin{array}{c} \begin{matrix} D & y' & w & w' & x' & y & z' & z \end{matrix} \\ \begin{bmatrix} | & \cdot & \cdot & \cdot & | & | & | & | \\ \cdot & | & \cdot & \cdot & | & \cdot & | & | \\ \cdot & \cdot & | & \cdot & \cdot & \cdot & | & | \\ \cdot & \cdot & \cdot & | & \cdot & | & \cdot & | \end{bmatrix} \end{array}$$



Example I (Cont'd.) Fig. 5



$f_1 = w'x'y' + w'x'z' + wxy'z' + wxyz$

$f_2 = w'x'y' + w'x'yz' + wxy'z' + wxyz$

$H_1 =$

	w'	w	x'	x	y'	y	z'	z	D
1	1	·	1	·	1	·	·	·	1
2	1	·	1	·	·	·	1	·	1
3	·	1	·	1	1	·	1	·	1
4	·	1	·	1	·	1	·	1	1
123	·	1	·	1	·	·	·	·	1

wx

$H_2 =$

	w'	w	x'	x	y'	y	z'	z	D
1	1	·	1	·	1	·	·	·	1
2	1	·	1	·	·	1	1	·	1
3	·	1	·	1	1	·	1	·	1
4	·	1	·	1	·	1	·	1	1
123	·	1	·	1	·	1	·	·	1

wxy

	y_1'	y_2'
1	1	·
3	·	1

(a)

y_1'	y_2'
1	1
·	1

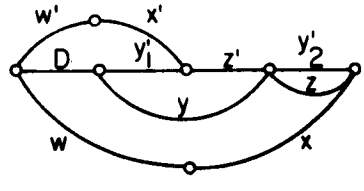
(b)

y_1'	y_2'
1	·
1	1

(c)

$L_{M2_C} =$

y	(w'x')	(wx)	z	D	y_1'	z'	y_2'
1	·	·	·	·	1	1	·
·	1	·	·	1	1	·	·
·	·	1	·	1	1	1	1
·	·	·	1	·	·	·	1



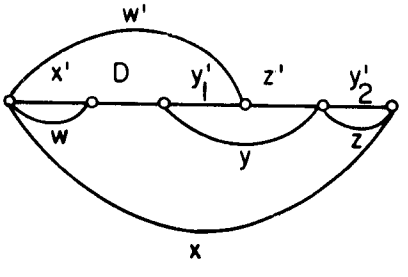
Example 2 - Fig. 6

$H_3 =$

	y	w'	x'	w	x	z	D	y'_1	z'	y'_2	
1	1	·	·	·	·	·	·	1	1	·	
2	·	1	1	·	·	·	1	1	·	·	
3	·	·	·	1	1	·	1	1	1	1	
4	·	·	·	·	·	1	·	·	·	1	
5	·	·	1	1	·	·	·	·	·	·	wx'

$L_{M3} =$

	y	w'	x	z	w	x'	D	y'_1	z'	y'_2
1	1	·	·	·	·	·	·	1	1	·
2	·	1	·	·	·	1	1	1	·	·
35	·	·	1	·	·	1	1	1	1	1
4	·	·	·	1	·	·	·	·	·	1
5	·	·	·	·	1	1	·	·	·	·



Example 2 (Cont'd.) Fig. 7

$$T_{12} = abc + ace + ag + bcdf + cef + fg$$

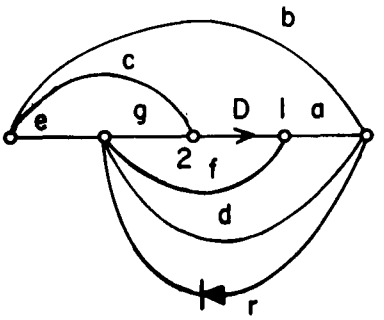
$$T_{21} = abc + abeg + acde + adg + bcf + cef + fg$$

$$F = [abc + cef + fg] + [aced + adg + abeg + bcdf] + acer + agr + bcfr$$

$$H = \begin{array}{c} \begin{array}{cccccccccc} a & b & c & d & e & f & g & r & D & \end{array} \\ \left[\begin{array}{cccccccccc} 1 & 1 & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \\ \cdot & \cdot & \cdot & \cdot & \cdot & 1 & 1 & \cdot & 1 & \\ \cdot & \cdot & 1 & \cdot & 1 & 1 & \cdot & \cdot & 1 & \\ 1 & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & 1 & 1 & \\ 1 & \cdot & 1 & \cdot & 1 & \cdot & \cdot & 1 & 1 & \\ \cdot & 1 & 1 & \cdot & \cdot & 1 & \cdot & 1 & 1 & \\ 1 & \cdot & \cdot & 1 & \cdot & \cdot & 1 & \cdot & 1 & \\ 1 & 1 & \cdot & \cdot & 1 & \cdot & 1 & \cdot & 1 & \\ 1 & \cdot & 1 & 1 & 1 & \cdot & \cdot & \cdot & 1 & \\ \cdot & 1 & 1 & 1 & \cdot & 1 & \cdot & \cdot & 1 & \end{array}\right] \end{array}$$

$$L_M = \begin{array}{c} \begin{array}{cccccccccc} c & r & d & f & b & a & e & g & D & \end{array} \\ \left[\begin{array}{cccccccccc} 1 & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & 1 & \cdot & 1 \\ \cdot & 1 & \cdot & \cdot & \cdot & 1 & \cdot & 1 & 1 & 2 \\ \cdot & \cdot & 1 & \cdot & \cdot & 1 & \cdot & 1 & 1 & 3 \\ \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & 1 & 1 & 4 \\ \cdot & \cdot & \cdot & \cdot & 1 & 1 & 1 & 1 & 1 & 5 \end{array}\right] \end{array}$$

$$\begin{array}{r} 2\text{-}1 \\ 1\text{+}2\text{-}4\text{-}5 \end{array} \left[\begin{array}{cccccccccc} -1 & +1 & \cdot & \cdot & \cdot & +1 & -1 & \cdot & +1 & \\ +1 & +1 & \cdot & -1 & -1 & \cdot & \cdot & \cdot & -1 & \end{array}\right]$$



Example 3 Fig. 8

$$T_{12} = abc + ac + ae + cd + de$$

$$T_{21} = abc + abe + bcd + de$$

$$F = [abc + de] + [abe + bcd] + [ac + ae + cd]\, r$$

$$H = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ \\ 234 \\ 235 \\ 245 \end{array} \begin{array}{c} \begin{array}{ccccccc} a & b & c & d & e & r & D \end{array} \\ \left[\begin{array}{ccccccc} 1 & 1 & 1 & \cdot & \cdot & \cdot & 1 \\ \cdot & \cdot & \cdot & 1 & 1 & \cdot & 1 \\ 1 & \cdot & \cdot & \cdot & 1 & 1 & 1 \\ 1 & \cdot & 1 & \cdot & \cdot & 1 & 1 \\ \cdot & \cdot & 1 & 1 & \cdot & 1 & 1 \\ 1 & 1 & \cdot & \cdot & 1 & \cdot & 1 \\ \cdot & 1 & 1 & 1 & \cdot & \cdot & 1 \\ \\ \cdot & \cdot & 1 & 1 & \cdot & \cdot & 1 \\ 1 & \cdot & 1 & \cdot & \cdot & \cdot & 1 \\ 1 & \cdot & \cdot & \cdot & 1 & \cdot & 1 \end{array}\right] \end{array} \begin{array}{l} \\ \\ \\ \\ \\ x \\ x \end{array}$$

$$\begin{array}{ccc} \begin{array}{cc} r_1 & r_2 \end{array} & \begin{array}{cc} r_1 & r_2 \end{array} & \begin{array}{cc} r_1 & r_2 \end{array} \\ \left[\begin{array}{cc} 1 & 1 \\ 1 & \cdot \\ \cdot & 1 \end{array}\right] & \left[\begin{array}{cc} 1 & \cdot \\ 1 & 1 \\ \cdot & 1 \end{array}\right] & \left[\begin{array}{cc} 1 & \cdot \\ \cdot & 1 \\ 1 & 1 \end{array}\right] \\ (a) & (b) & (c) \end{array}$$

$$\left[\begin{array}{cc} \cdot & 1 \\ 1 & \cdot \\ 1 & 1 \end{array}\right] \quad \left[\begin{array}{cc} \cdot & 1 \\ 1 & 1 \\ 1 & \cdot \end{array}\right] \quad \left[\begin{array}{cc} 1 & 1 \\ \cdot & 1 \\ 1 & \cdot \end{array}\right] \begin{array}{l} 234 \\ 235 \\ 245 \end{array}$$

Example 4 Fig.9

$L_{M1} =$

	b	d	e	c	a	r_1	r_2	D	
1	1	·	·	·	·	1	·	·	
2	·	1	·	·	1	1	1	·	
3	·	·	1	·	1	1	1	1	a e r_1 r_2 (T_{12})
4	·	·	·	1	1	1	·	1	a c r_1 (T_{12})
124	+1	−1		+1		+1	−1	+1	b c d r_1 r_2

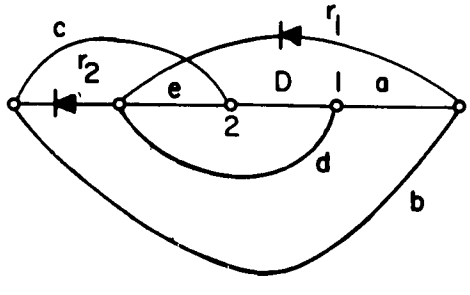
Results from (a)

$L_{M2} =$

	b	d	r_1	c	r_2	a	e	D	
1	1	·	·	·	1	1	1	1	
2	·	1	·	·	·	·	1	1	
3	·	·	1	·	·	1	1	1	
4	·	·	·	1	1	·	1	·	
123	+1	+1	−1		+1		+1	+1	b d e r_1 r_2
14	+1	·	·	−1	·	+1	·	+1	a b c
24	·	+1	·	−1	−1	·	·	+1	c d r_2
34	·	·	+1	−1	−1	+1	·	+1	a c r_1 r_2
1234	+1	+1	−1	−1	·	·	·	+1	b c d r_1

Results from (b)

From L_{M2}



Example 4 (Cont'd)

Fig. 10

$$T_{12} = x'y'z' + x\,y\,z'$$

$$T_{21} = x\,y\,z + x'\,y\,z$$

$$F = x'y'z' + x\,y\,z'r + x'y\,z\,r$$

$H =$

	x	x'	y	y'	z	z'	r	D
1	1	·	1	·	·	1	1	1
2	·	1	1	·	1	·	1	1
3	·	1	·	1	·	1	·	1
123	1	·	·	1	1	·	·	1

$L_1 =$

	x	x'_1	x'_2	y	y'	z	z'	r	D	
	1	·	·	1	·	·	1	1	1	1
	·	1	·	1	·	1	·	1	1	2
	·	·	1	·	1	·	1	·	1	3
	1	1	1	·	1	1	·	·	1	123

$L_2 =$

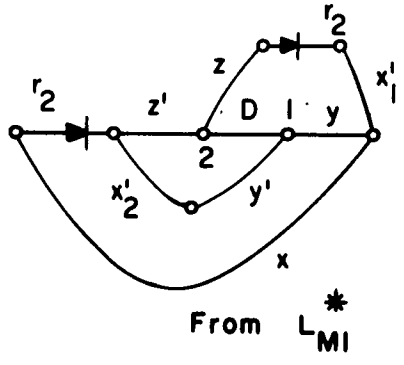
x	z	y'	x'	z'	y	r_1	r_2	D
1	·	·	·	1	1	1	·	1
·	1	·	1	·	1	·	1	1
·	·	1	1	1	·	·	·	1

$L_{MI} =$

x	$(x'_1\ z)$	$(x'_2\ y')$	y	z'	r	D
1	·	·	1	1	1	1
·	1	·	1	·	1	1
·	·	1	·	1	·	1

$L^*_{MI} =$

x	$(x'_1\ z)$	$(x'_2\ y')$	y	z'	r_1	r_2	D
1	·	·	1	1	1	·	1
·	1	·	1	·	·	1	1
·	·	1	·	1	·	·	1



Example 5 — Fig. 11

REFERENCES

1. Hlgonnet, R. A., and Grea, R.A., Logical Design of Electrical Circuits, New York, Mc Graw-Hill, 1958.
2. Mc Cluskey, E.J., Detection of Group Invariance or Total Symmetry of a Boolean Function, Bell System Tech. J. 35, 1445-1453 (1956).
3. Caldwell, S.H., Switching Circuits and Logical Design, New York, Wiley, 1958.
4. Shannon, C.E., The Synthesis of Two-Terminal Switching Circuits, Bell System Tech. J. 28, 59-98 (1949).
5. Scheinman, A.H., A Numerical-Graphical Method for Synthesizing Switching Circuits, Trans. AIEE (Commun. and Electronics) 34, 687-689 (1958).
6. Roginskij, V.N., A Graphical Method for the Synthesis of Multiterminal Contact Networks, Proc. Internatl. Symp. Theory of Switching, Harvard Univ., Cambridge, Mass., Pt. 2, 302-315 (1957).
7. Hohn, F.E., and Schissler, L.R., Boolean Matrices in the Design of Combinational Switching Circuits, Bell System Tech. J. 34, 177-202 (1955).
8. Semon, W., Matrix Methods in the Theory of Switching, Proc. Internatl. Symp. Theory of Switching, Harvard Univ., Cambridge, Mass., Pt. 2, 13-50 (1957).
9. Okada, S., Topology Applied to Switching Circuits, Proc. Polytechnic Inst. Brooklyn Symposium on Information Networks 3, 267-290 (1954).
10. Gould, R., Application of Graph Theory to Synthesis of Contact Networks, Proc. Internatl. Symp. Theory of Switching, Harvard Univ., Cambridge, Mass., Pt. 2, 244-292 (1957).
11. Mayeda, W., Synthesis of Switching Functions by Graph Theory, IBM Jl. Res. and Devel. 3, 321-328 (1960).
12. Okada, S., and Young, K.P., Ambit Method of Realizing Cut-Set Matrices, Polytechnic Institute of Brooklyn, Microwave Research Institute, Research Report PIBMRI-801-60 (1960).
13. Moriwaki, Y., Further Contributions to the Realization of Boolean Polynomials Based on Incidence Matrices, Ibid., Research Report PIBMRI-938-61 (1961).
14. Ashenhurst, R.L., The Theory of Abstract Two-Terminal Switching Networks (The Theory of Switching) Harvard Computation Laboratory, Report No. BL-10, 1-8 (1955).
15. Seshu, S. and Reed, M.D., Linear Graphs and Electrical Networks, Reading, Mass., Addison-Wesley (1961).
16. Gould, R., Graphs and Vector Spaces, J. Math. and Phys. 38, 193-214 (1958).
17. Lofgren, L., Solution to the Realizability Problem for Irredundant Boolean Branch-Networks, J. Franklin Inst. 268, 352-377 (1959).
18. Berge, C., Theorie Des Graphs Et Ses Applications, Paris, Dunod, 1958.